On the Effect of a Shoulder Belt in a Collision Involving a Longitudinal Deceleration

The present paper presents a method of calculating the force of a lap belt and shoulder belt on the wearer in the case of a collision involving a longitudinal deceleration; that is, a deceleration in the direction of the longitudinal axis of the vehicle, as would be the case in a head-on collision. Three separate cases are considered: First, the case of a vehicle occupant wearing only a lap belt; second, the case of a vehicle occupant wearing a lap belt and shoulder belt, with the shoulder belt locked the time of the onset of the collision; and third, the case of a vehicle occupant wearing a lap belt and shoulder belt with a time delay between the onset of collision and the time the shoulder belt locks.

Case 1. Lap belt only. We regard the head and torso as a single unit. A clearer understanding of the equations may be had by referring to figure 1.

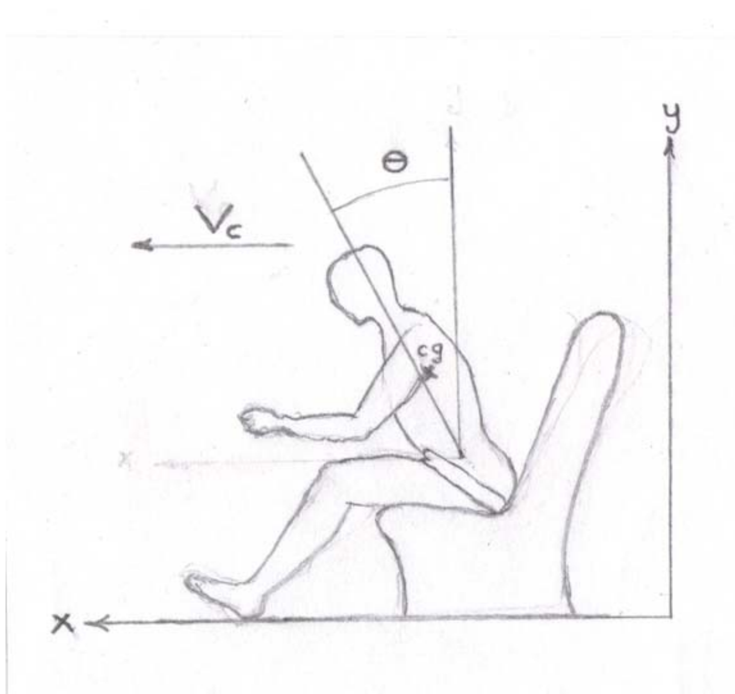


Figure 1

The x-y coordinate system is Newtonian; that is, the x-y coordinate system is stationary and the vehicle is moving with respect to the x-y coordinate system. Notice that we are using a left handed coordinate system.

Taking forces and moments about the cg of the upper body (head and torso), we get three equations with three unknowns:

(1) $$F_{0y} r_{cg} \sin\theta - F_{0x} r_{cg} \cos\theta = I_{cg} \ddot{\theta}$$

(2) $$F_{0x} = m\left(\ddot{\theta} r_{cg} \cos\theta - \dot{\theta}^2 r_{cg} \sin\theta + \dot{V}_c\right)$$

(3) $$F_{0y} = W - m\left(r_{cg}\ddot{\theta}\sin\theta + r_{cg}\dot{\theta}^2\cos\theta\right)$$

where F_{ox} represents the x component of the force of the lap belt on the upper body of the wearer, F_{oy} represents the y component, θ the angle which the upper body makes with the vertical, I_{cg} the moment of inertia about the cg of the upper body, m the mass of the upper body, W the weight (=mg), $\dot{V}_c$ the rate of change of the velocity of the vehicle, and r_{cg} the distance from the waist to the center of mass of the upper body.

The three unknowns are F_{ox}, F_{oy}, and θ. Substituting for F_{ox} and F_{oy} from equations 2 and 3 into equation 1, we obtain a single second order, linear differential equation for θ of the form,

(4) $$\ddot{\theta} = A\sin\theta + B\cos\theta$$

where

(5) $$A = g r_{cg} m / \left(I_{cg} + m r_{cg}^2\right) \quad \text{and} \quad B = A\dot{V}_c / g$$

This is a marching problem with the initial conditions at t=0, θ=0 and $\dot{\theta} = 0$. Equation 4 actually has a closed form solution of the form

(6) $$\dot{\theta} = \left[2A\left(\cos\theta_1 - \cos\theta\right) + 2B\left(\sin\theta - \sin\theta_1\right)\right]^{1/2}$$

Since we wish to know $\theta, \dot{\theta}, and \ddot{\theta}$ as a function of time, however, we will integrate equation 4 numerically. The initial conditions are, $\theta = \theta_1, t = 0, \dot{\theta} = 0$. We can thus find $\ddot{\theta}_1$ from equation (4) and, choosing a value of δt , integrate forward:

(7) $$\dot{\theta}(n) = \dot{\theta}(n-1) + \ddot{\theta}(n-1)\delta t$$

and

(8) $$\theta(n) = \theta(n-1) + \left(\dot{\theta}(n-1) + \dot{\theta}(n)\right)\delta t / 2$$

Once we have θ as a function of time, we may substitute back in equations 1 and 2 to find the force of the lap belt on the wearer.

In order to integrate the equations we have written a short computer program (Appendix 1) and run a sample solution. The program is written in such a way that the parameters may be adjusted to suit any particular case. For the sample solution we have assumed that the seatbelt wearer was an average male in good physical condition, because the physical properties required, I_{cg}, m, and r_{cg}, were available in the open literature (1). In addition, we took the dummy test scenario, that is, 30 mph with a foot of crush at the front of the vehicle. Even though such accidents seldom happen, this scenario has been touted by NHTSA as demonstrating the virtues of seatbelts, so this was a good opportunity to see how valid NHTSA's claims really are. Since, according to the government's own data (2), the average vehicle involved in a fatal head-on collision was travelling at 56 mph, not 30 mph, the forces involved in such an accident are likely to be higher than those found in the present case. ♣

In order to find the total force of the lap belt on the wearer we need to add the force caused by the lower body. We exclude the mass of the legs below the knee because their forward motion would be limited by the back of the front seat in the case of a rear seat occupant, and by the firewall and dashboard in the case of a front seat occupant. Likewise, we exclude the arms, because it is not clear where the arms would be located at the time of collision. The most likely scenario would be that the occupant, seeing the crash coming, would grasp the back of the front seat or the dashboard so that the arms would already be forward at the time of collision. The mass and moments of inertia of the arms and lower legs are small compared to those of the rest of the body (1), so that ignoring these members will not make much difference. ♦

For the sample solution we took a value of δt =0.001 seconds and a total time of .045 seconds, since that is the time the vehicle takes to come to a stop in the present case. A typical car seatback is inclined at an angle of about 20 degrees to the vertical so we took an initial value of –20 degrees for theta. All of these parameters may be chosen at random for the computer program, so that the operator may choose them to suit any particular case.

The force contribution of the lower body of the lower body is given by

(9) $$F_l = m_l \dot{V}_c$$

♣ At 56 mph, the front seat occupants would be crushed to death as the vehicle is stove in, so the force of the seatbelts on the occupants would be more relevant on the rear seat occupants than on the front seat ones, assuming that these are not also crushed to death. That would depend on the particular type of vehicle and the other factors involved in the accident. According to data published by the NHTSA (2,3), rear seat occupants make up only about 10 percent of vehicle occupants and head-on collisions make up only about 2% of all collisions and fewer than 14% of fatal collisions.

♦ We actually did some tests to see how much force a vehicle occupant could exert on the back of the front seat or the dash with his hands and arms. It obviously depends on the vehicle occupant. We found the force to be in a range of between 40 and 80 pounds. Even recognizing the fact the people in emergencies can exert greater force, the order of magnitude of these forces is negligible compared to the force exerted by the seatbelt.

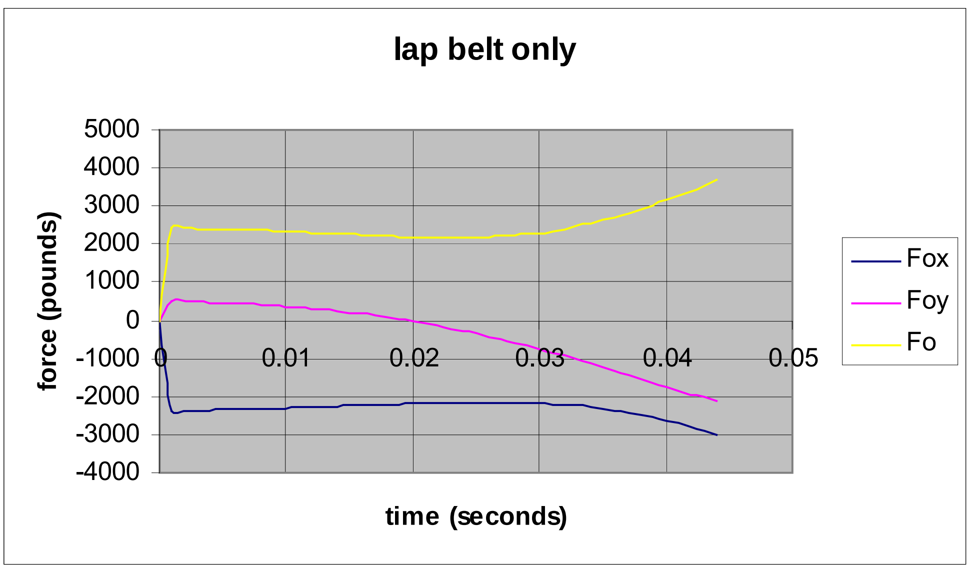
in the minus x direction, so that the total force of the lap belt on the wearer in the minus x direction becomes

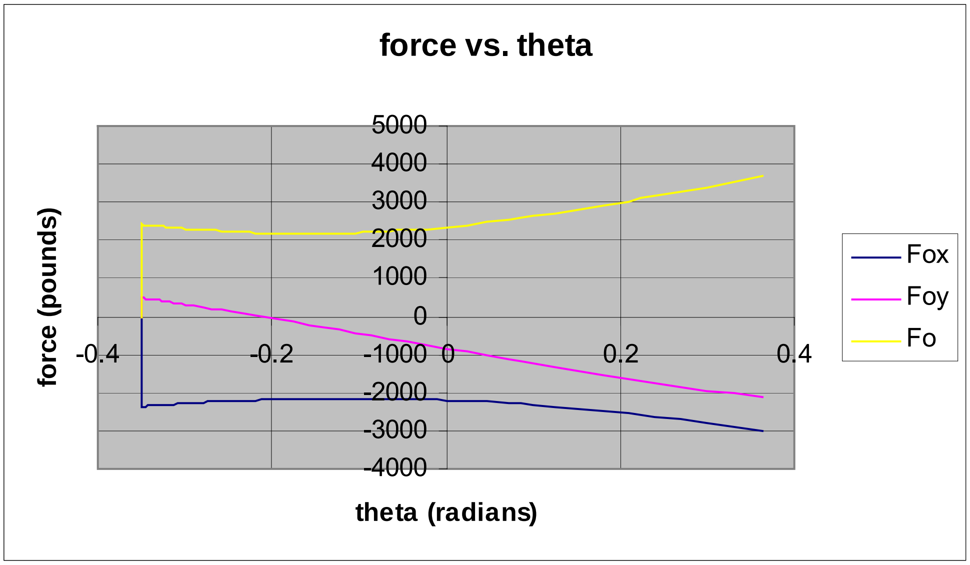
(10) $$F_{x0} = F_{0x} + F_l$$

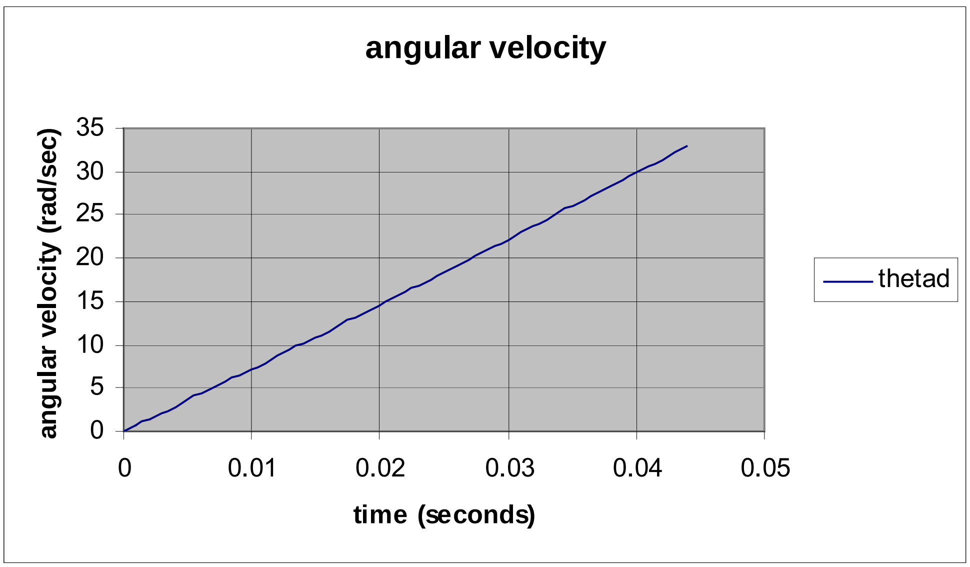
The total force of the lap belt on the wearer is given by

(11) $$F_0 = \left(F_{x0}^2 + F_{0y}^2\right)^{1/2}$$

The results of the calculations are given in table 1 (Appendix 1) and shown on the graphs below. Comparing the results for $\dot{\theta}$ from equations (6) and (7) in table 1 (thetad and thetd) we see almost no difference, indicating the accuracy of the numerical solution.







Looking at the results we see that the force of the lap belt on the wearer varies from an initial value of 2,414 pounds to a final value of 3,682 pounds, with a minimum value of 2,168 pounds. As regards the time period during which the force is applied, at 44 fps the vehicle travels two feet in 0.045 seconds. The time period is thus long enough for the seatbelt to cut the wearer in two. While this seldom happens, there is at least one recorded of a woman being cut in two by her lap belt (4). Usually, the lap belt kills the wearer by bursting his intestines and/or breaking his spleen.

Since the stomach cannot withstand forces of this magnitude it is clear that the intestines and intestinal muscles must be compressed until the deceleration rate of the body reaches that of the vehicle, unless either the belt breaks, or the vehicle stops, or wearer is cut in two. Since the belt is supposed to withstand a force of 6,000 pounds, according to NHTSA regulations (5) it is unlikely to break, even though the belt breaking or the anchor failing would be the only hope the wearer would have in such a situation of saving his life. In the present case the vehicle travels one foot prior to stopping, which is more than the thickness of the body of most vehicle occupants. The seatbelt wearer would thus stop in a somewhat longer distance than the vehicle, but no more than the sum of the vehicle stopping distance and the thickness of the body, unless the belt cuts the body in two. This means that the force of the seatbelt on the wearer would initially be less than the above calculated numbers until the stomach is compressed to the point where the resistance of the body is such that it does not compress further and thus decelerates at the same rate as the vehicle. At that point, the magnitude of the force would be that given by the above method of solution. The effect is somewhat like putting a 2,000 pound weight on a person's stomach.

Looking at the results in table 1 we find that the angle of inclination of the upper body is 21 degrees at the time the vehicle stops, with an angular velocity of l874 degrees per second, corresponding to a velocity of the head of 62 feet per second for a distance of 1.9 feet from the waist to the center of gravity of the head, based on the measurements of our model. It is likely, therefore, that a rear seat passenger would hit his head on the back of the front seat with a considerable velocity, but considering that the passenger would already have suffered fatal injuries from his seatbelt , it would simply add to his injuries. Since the front seatback is padded, it would lessen the force of the impact on his head. There can be no question of a rear seat passenger "jack-knifing", as has sometimes been alleged, since the front seat is in the way, unless the passenger is a small child or a midget. An average front seat passenger in an average vehicle would likely not hit his head on the dashboard, but this would depend on the height of the passenger and the interior design of the vehicle. The driver would hit his chest on the steering wheel.

Case 2. We now consider the same type of collision, but with the seatbelt wearer wearing a shoulder belt which is locked at the time of collision. In this case, the torso is held rigid but the head is free to move forward. The situation is illustrated in figure 2. As in the first case (figure 1), the x-y coordinate system is Newtonian and the vehicle is moving with respect to the x-y coordinate system.

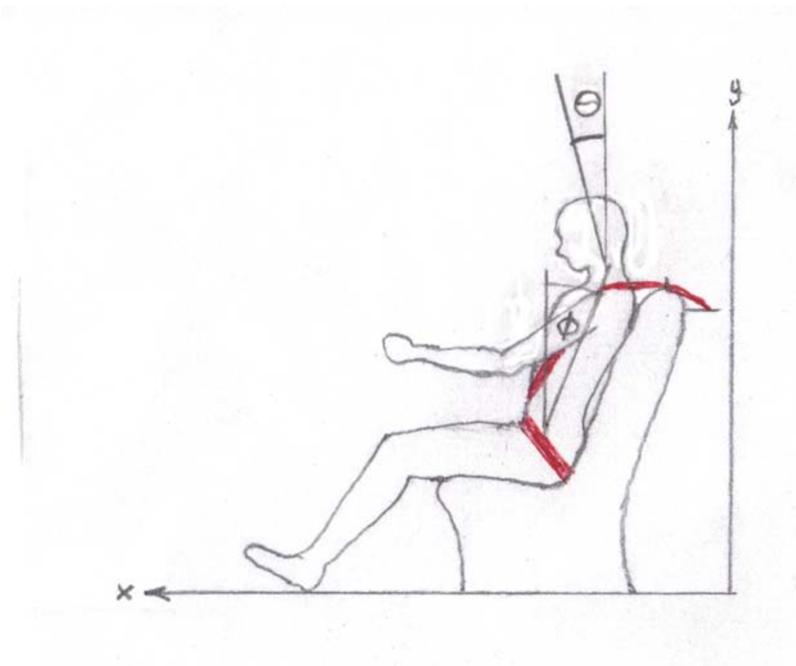


Figure 2

The system of equations for the head is basically the same as the system of equations for the upper body given in the previous case, except that the equations now apply to the head only. Thus we have

(12) $$F_{1xh} = m_h\left(-r_{cgh}\dot{\theta}^2\sin\theta + r_{cgh}\ddot{\theta}\cos\theta - \dot{V}_c\right)$$

(13) $$F_{1yh} - m_h g = m_h\left(-r_{cgh}\dot{\theta}^2\cos\theta - r_{cgh}\ddot{\theta}\sin\theta\right)$$

(14) $$F_{1xh}r_{cgh}\cos\theta + F_{1yh}r_{cgh}\sin\theta = I_{cgh}\ddot{\theta}$$

where the forces F_{1xh} and F_{1yh} are now exerted at the neck, m_h is the mass of the head, and r_{cgh} the distance from the neck to the center of gravity of the head. Thus,

(16) $$\ddot{\theta} = A\sin\theta + B\cos\theta$$

as before, except that A and B are now given by

(15) $$A = \left(m_h r_{cgh} g\right)/\left(m_h r_{cgh}^2 + I_{cgh}\right) \quad \text{and} \quad B = A\dot{V}_c / g$$

where I_{cg} is the moment of inertia of the head about its center of gravity.

Since the torso is now fixed by the shoulder belt., the moment about the center of gravity of the torso must be zero. Since the angle of the torso will be negative with respect to the vertical (angle of seat back) the shoulder belt acts roughly on the center of gravity of the torso so that its contribution to the moment about the center of gravity of the torso will be small and may be neglected. Thus,

(16) $$F_{0x} r_{cgt} = F_{1x}\left(r_1 - r_{cgt}\right)$$

Solving for F_{1x} by the same method as in the previous case, we may solve for Fox from equation (16). The force of the shoulder belt on the wearer may then be found from the equation

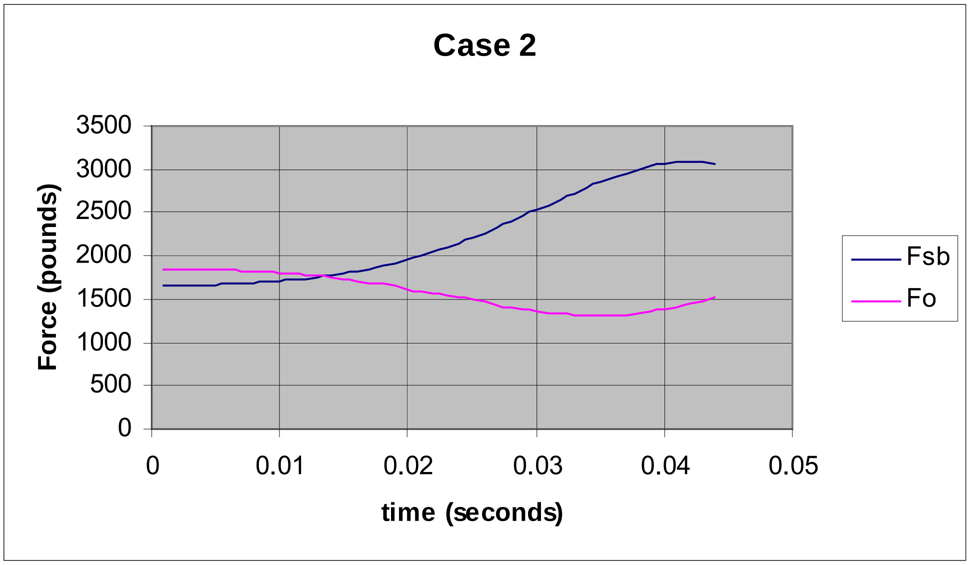
(17) $$F_{sbx} + F_{0x} + F_{1tx} = m_t \dot{V}_c$$

where F_{1xt} = -$F_{1xh.}$ For the case where $\phi = 0$, $F_{sb} = F_{sbx}$. Since the force of the shoulder belt on the torso will be essentially perpendicular to the torso, when Φ is less than or equal to 0 we can find F_{sb} from F_{sbx} by the relation

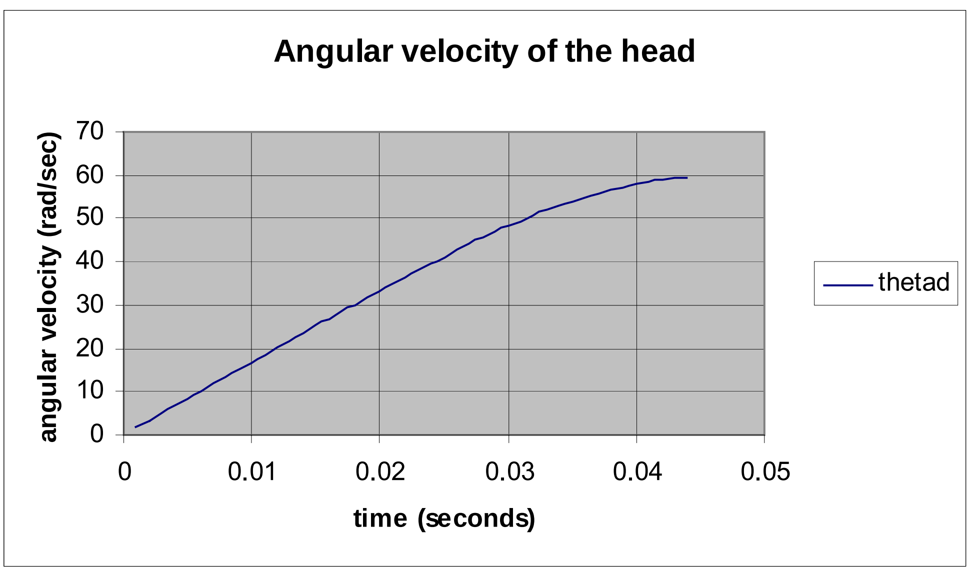
(18) $$F_{sb} = F_{sbx} / \cos\phi$$

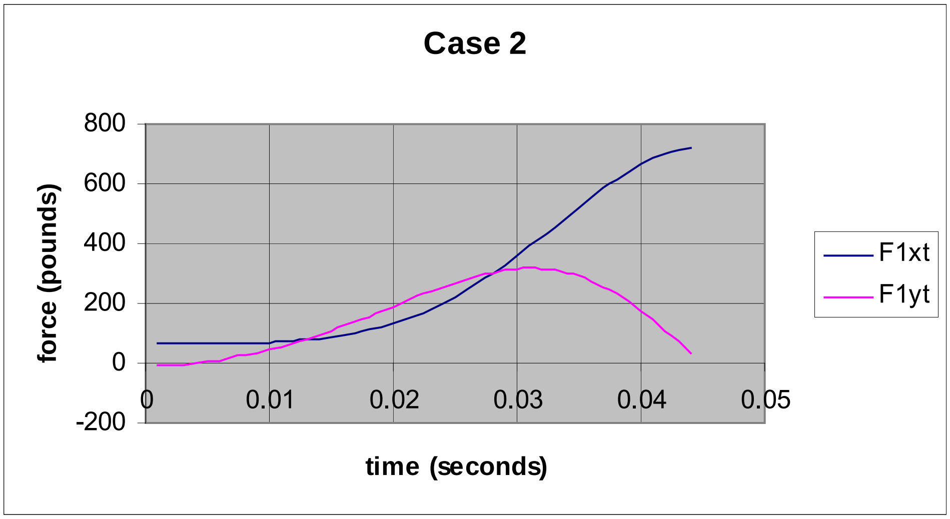
where Φ is the angle the torso makes with the vertical, with counterclockwise taken as positive.

In order to solve the equations we write short computer program (Appendix 2). We present a sample calculation for the case where Φ = -20 degrees and the initial angle of the head, $\theta_1 = 0$. (Note that θ_1 and Φ may be chosen as desired.) The results are shown in table 2, Appendix 2, and on the graphs below. The force of the shoulder belt on the wearer varies from 1,659 pounds to 3,070 pounds and the force of the lap belt varies from 1,852 pounds to 1,517 pounds. Thus, while the presence of the shoulder belt decreases the force on the lap belt, it does not decrease it enough to prevent severe or fatal injury to the wearer. At the same time, the force of the shoulder belt on the chest is sufficient to crush the chest, adding an additional severe or fatal injury. While data is not available on how much force would be needed on the chest and stomach to cause fatal injury, the magnitude of these forces is such to cause almost certain death. For examples of this happening in real accidents, see references (4) and (6).



Another factor which must be considered is the angular velocity of the head, commonly known as whiplash. In the present case, we find the inclination of the head with the vertical of 86 degrees when the car stops, or 116 degrees with respect to the torso. For most people, the maximum angle of inclination of the head with respect to the torso without breaking the neck is less than 116 degrees, usually closer to 45 degrees. The angular velocity of the head when the car stops is found to be 59 radians per second and about 38 radians per second when the head is inclined at an angle of 45 degrees with the torso. Again, data is lacking on how much angular velocity it would take to break the neck, but there is at least one recorded case of a person's spine being severed at the neck in this type of accident (6). It is likely, therefore, that such whiplash would prove fatal.





Thus we see that in the case of a shoulder belt locked at the time of collision the wearer is likely to suffer three fatal injuries instead of just one: burst intestines, crushed chest and broken neck.

Case 3. Shoulder belt locks after time delay. With the type of shoulder belt currently in use, a finite interval will occur between the onset of collision and the time the belt locks. Locking mechanisms currently in use are of two types: inertial and explosive. We conducted tests on a l979 Ford and 2004 KIA, both with inertial type locks, to see if we could get the belt to lock. On the l979 Ford we were not able to get the belt to lock at all. On the 2004 KIA we could only get the belt to lock once in ten tries, and then only after the belt had extended about a foot. The fact that this type of locking mechanism was not considered satisfactory by NHTSA is indicated by the fact that newer model cars have so-called seatbelt pretensers. These consist of explosive charges which are, in theory, set off by a computer under certain criteria, such as a rapid longitudinal deceleration. The explosive is supposed to trip the locking mechanism, locking the belt. As our previous research has shown, (7), there is a time delay between the onset of collision and the firing of the explosive, which is, typically, around 21 milliseconds. Up to the time the belt locks, the situation would be the same as in case 1, as the drag of the unlocked shoulder belt is negligible compared to the forces involved. When the locking mechanism engages, the rate of rotation of the torso would go from roughly 16 radians per second (case 1) to zero almost instantaneously. If the stop in rotation were instantaneous, the force of the shoulder belt on the wearer would be infinite. Since the chest cannot withstand an infinite force, the force on the chest would rise only to the point where the chest would be crushed.

We can get some idea of this force for a given amount of crush by making certain simplifying assumptions. We initially neglect the force of the head on the torso, since this force will be small compared to the force of the seatbelt (see previous section). This force may be calculated a posteriori and a more exact solution obtained by an iterative process. We do not know the rate of crush of the chest as a function of time. It is likely that, in reality, the force of the shoulder belt on the chest would rise to the point where the ribs would crack, whereupon the force, meeting less resistance, would drop and the rate of penetration would rise as the broken ribs are driven into the heart and lungs. Instead, we shall assume an average rate of penetration. This will give us a lower maximum force as the maximum force is smallest when the rate of penetration is constant. The situation is shown in figure 3:

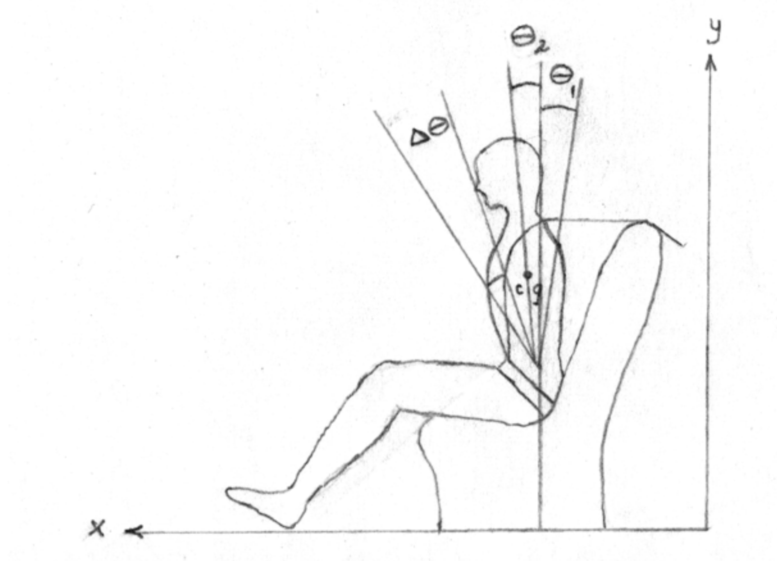


Figure 3

The degree of penetration will be $r_{cg}\ \Delta\theta$. We may calculate the force for any value of $r_{cg}\,\Delta\theta$. Take the initial value of θ as θ_1 and the final value as θ_2 . Then $\Delta\theta = \theta_2 - \theta_1$. Up to the time the belt locks, dθ/dt is given by the solution from case 1. Thus we can find the solution for any value of θ_1. The average dθ/dt during the time the chest is being crushed will be taken as $(d\theta/dt)_1/2$ since $(d\theta/dt)_2 = 0$. The time Δt of crush will thus be $\Delta t = 2\Delta\theta/(d\theta/dt)_1$. The average $d^2\theta/dt^2$ will thus be

(19) $$\ddot{\theta} = (\theta_2 - \theta_1)/(t_2 - t_1) = -\dot{\theta}_1 / \Delta t$$

since $(d\theta/dt)_2 = 0$. It will thus be observed that if there were no penetration, Δt would be zero and $d^2\theta/dt^2$, and hence the force on the chest, would be infinite. Since the chest cannot withstand an infinite force it is clear that there must be some degree of penetration. If the force were small, this could take the form of a contraction of the chest. But, as we shall see, the force is not small in even a moderate collision, such as the dummy test scenario.

The velocity of the center of gravity of the chest. V_{cg}, will be given by

(20) $$\vec{V}_{cg} = \vec{V}_c + \vec{r}_{cg} x \vec{\dot{\theta}}$$

It follows that

(21) $$V_{cgx} = V_c + \dot{\theta} r_{cg} \cos\theta$$

(22) $$-V_{cgy} = r_{cg} \dot{\theta} \sin\theta$$

Differentiating with respect to t, we get

(23) $$\dot{V}_{cgx} = a_c + r_{cg} \ddot{\theta} \cos\theta - r_{cg} \dot{\theta}^2 \sin\theta$$

(24) $$\dot{V}_{cgy} = r_{cg} \ddot{\theta} \sin\theta + r_{cg} \dot{\theta}^2 \cos\theta$$

Taking forces and moments for the torso,

(25) $$F_{0xt} + F_{sb} \cos\theta = m_t \dot{V}_{cgx}$$

(26) $$F_{0yt} + F_{sb} \sin\theta = m_t \dot{V}_{cgy}$$

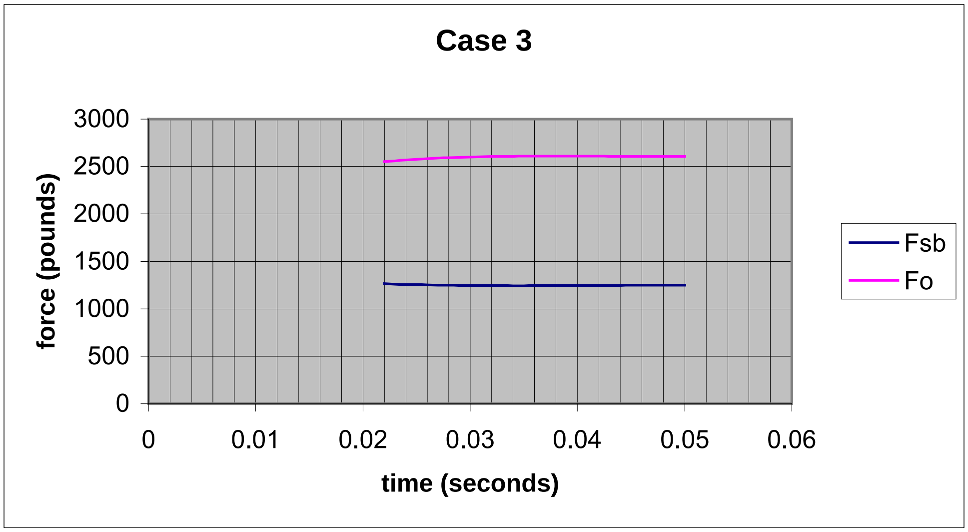
(27) $$F_{0x} r_{cgy} + F_{0y} r_{cgx} = I_{cg} \ddot{\theta}$$

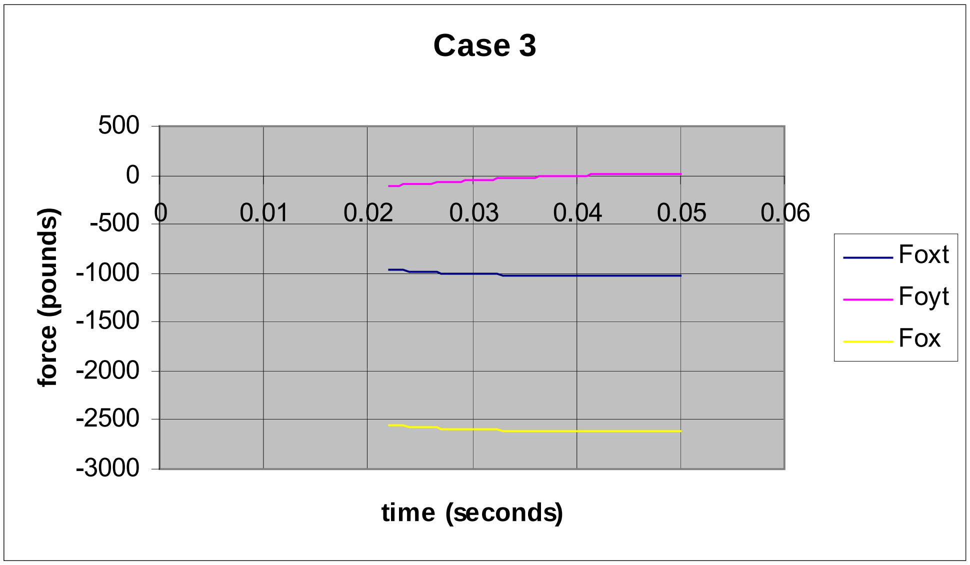
since for the case θ less than or equal to zero the center of pressure of the shoulder belt is essentially on the c.g. of the torso. Since dθ/dt and $d^2\theta/dt^2$ are known, we can find θ(t):

(28) $$\theta(t+\delta t) = \theta(t) + \dot{\theta}(t)\delta t + \ddot{\theta}(t)\delta t^2 / 2$$

Hence we have three equations, (25), (26), and (27), and three unknowns, F_{ox}, F_{0y} and F_{sb}.

To solve the equations we have written a short computer program and used it to generate a sample solution (Appendix 3). We take t_1 = 0.021 seconds which corresponds to the likely time delay of the seatbelt pretensers ( ). What it would be for an inertial type of lock would depend on the particular belt and lock as well as on the shape, weight and size of the vehicle occupant. Using the results of case 1 for an initial value of θ = -0.35, we find at t = 0.021 that θ_1 = -0.19 radians and $(d\theta/dt)_1$ = 16.21 radians per second. Assuming that the chest is crushed to a depth of two inches, we find that Δt = 0.03 seconds. Thus we integrate equations (25) – (28) up to t = 0.05 seconds. Since in the present case the vehicle stops at 0.045 seconds, the solution is only valid up to that point. The solution is presented in table 3 (Appendix 3) and the graphs below:





The solution shows that the maximum force of the shoulder belt on the chest is 1,264 pounds. The maximum value of θ is less than 3 degrees (at t = 0.045) and so falls within the range of validity of the solution to a good approximation.

While the shoulder belt reduces the force on the lap belt somewhat, the force is still large enough to cause fatal internal injuries to the wearer. Additional injuries would be caused by the whiplash of the head on the neck.

Conclusion: The calculations show that the presence of a shoulder belt would increase the number of injuries suffered by the wearer if the shoulder belt locks before the vehicle stops. The extent and nature of the injuries would depend on if and when the belt locks up. Given the erratic nature of inertia locks, there is no guarantee that the belt would lock up at all, in which case it would make little difference. The most likely outcome, if the belt locks before the car stops, is that the wearer would suffer three fatal injuries instead of just one: burst intestines, crushed chest and broken neck.

References:

1. Anthropometry and Mass Distribution for Human Analogues, Volume 1. Naval Biodynamics Laboratory NBDL-87R003, March l988. (This report is also available as Aerospace Medical Research Laboratory report no. AAMRL-TR-88-010, Wright-Patterson Air Force Base, United States Air Force)

2. Fatal Accident Reporting System (FARS) http://www-fars.nhtsa.dot.gov

3. Traffic Safety Facts http://www-nrd.nhtsa.dot.gov/pubs/tsf2006fe.pdf (note: this report is published every year)

4. http://www.safetychoice.org/newspapergallery/newspaper_gallery.htm

5. Federal Motor Vehicle Safety Standard (FMVSS) 208. Code of Federal Regulations (CFR) 49-571

6. Seatbelt Victims – the Book of the Dead. In memoriam. http://www.safetychoice.org/Documents/seatbeltvictims.pdf

7. Effect of System Delay on Air Bag Deployment. Air Bag Progress Report (abpr) 10. A Safety Choice Coalition technical report. http://www.safetychoice.org/Documents/Airbag Progress Report 10.pdf

APPENDIX 1

```
      program scourt
      integer n
      real m,Icg,g,rcg,Vcd,A,B,t(45),theta(45),thetad(45),thetdd(45),
     +thetd(45),dt,Fox(45),Foy(45),Fo(45),C(45),D(45),E(45),H(45),ml,
     +Fl
C  m: mass of upper body, in slugs
C  Icg: moment of inertia of upper body about the center of gravity
C       of the upper body, in slug-feet squared
C  g: acceleration due to gravity, in feet per second squared
C  rcg: distance from waist to center of gravity of the upper body,
C       in feet
C  Vcd: rate of deceleration of the vehicle, in feet per second squared
C  t: time in seconds
C  theta: angle upper body makes with the vertical, in radians
C  thetad: time rate of change of theta, in radians per second
C  thetdd: second derivative of theta with respect to time, in radians
C          per second squared
C  thetd: closed form solution for thetad, in radians per second
C  dt: time increment used in the numerical solution, in seconds
C  Fox: x component of the force of the lap belt on the wearer, in
C       pounds
C  Foy: y component of the force of the lap belt on the wearer, in
C       pounds
C  Fo: force of the lap belt on the wearer, in pounds
C  ml: mass of the lower body, in slugs
      m=2.0
      ml=1.64
      g=32.2
      Icg=0.676
      Vcd=968.0
      Fl=ml*Vcd
      rcg=0.86
      dt=0.001
      A=g*rcg*m/(Icg+m*rcg**2)
      B=A*Vcd/g
      t(1)=0
      theta(1)=-0.35
      thetad(1)=0
      thetdd(1)=A*SIN(theta(1))+B*COS(theta(1))
      DO 10 n=2,45
      thetad(n)=thetad(n-1)+dt*thetdd(n-1)
      theta(n)=theta(n-1)+dt*(thetad(n-1)+thetad(n))/2.0
      thetdd(n)=A*SIN(theta(n))+B*COS(theta(n))
      thetd(n)=SQRT(2.0*A*COS(theta(1))-2.0*A*COS(theta(n))+
     +2.0*B*SIN(theta(n))-2.0*B*SIN(theta(1)))
      t(n)=t(n-1)+dt
      C(n)=rcg*thetdd(n)*COS(theta(n))
      D(N)=(thetad(n)**2)*rcg*SIN(theta(n))
      Fox(n)=m*(C(n)-D(n)-Vcd)-Fl
      E(n)=g-rcg*thetdd(n)*SIN(theta(n))
      H(n)=rcg*(thetad(n)**2)*COS(theta(n))
      Foy(n)=m*(E(n)-H(n))
      Fo(n)=SQRT(Fox(n)**2+Foy(n)**2)
```

```
   10 CONTINUE
      PRINT 20
   20 FORMAT('',T4,'t',T8,'theta',T16,'thetad',T26,'thetd',
     +T35,'thetdd',T45,'Fox',T54,'Foy',T63,'Fo')
      DO 30 n=2,45
      PRINT 25, t(n),theta(n),thetad(n),thetd(n),thetdd(n),Fox(n),
     +Foy(n),Fo(n)
   25 FORMAT(F5.3,2X,F5.3,2X,F7.3,2X,F7.3,2X,F7.1,2X,F7.1,2X,F7.1,
     +2X,F7.1,2X,F7.1)
   30 CONTINUE
      END
```

Table 1

t	theta	thetad	thetd	thetdd	Fox	Foy	Fo
0.000	-.350	0.000	0.000	716.9	0.0	0.0	0.0
0.001	-.350	0.717	0.717	717.0	-2364.6	486.0	2414.0
0.002	-.349	1.434	1.434	717.3	-2362.8	482.5	2411.5
0.003	-.347	2.151	2.151	717.8	-2359.7	476.5	2407.3
0.004	-.344	2.869	2.870	718.5	-2355.4	468.2	2401.5
0.005	-.341	3.588	3.588	719.4	-2349.9	457.4	2394.0
0.006	-.337	4.307	4.308	720.6	-2343.4	444.2	2385.1
0.007	-.332	5.028	5.029	721.9	-2335.7	428.5	2374.7
0.008	-.327	5.749	5.751	723.3	-2327.1	410.2	2362.9
0.009	-.321	6.473	6.475	725.0	-2317.5	389.4	2350.0
0.010	-.314	7.198	7.201	726.8	-2307.1	365.9	2335.9
0.011	-.307	7.924	7.928	728.8	-2295.9	339.7	2320.9
0.012	-.298	8.653	8.658	730.9	-2284.1	310.7	2305.1
0.013	-.289	9.384	9.389	733.1	-2271.7	278.9	2288.8
0.014	-.279	10.117	10.123	735.5	-2259.0	244.1	2272.2
0.015	-.269	10.853	10.860	737.9	-2246.1	206.4	2255.6
0.016	-.258	11.591	11.598	740.5	-2233.1	165.6	2239.2
0.017	-.246	12.331	12.340	743.1	-2220.2	121.7	2223.6
0.018	-.233	13.074	13.083	745.7	-2207.7	74.6	2208.9
0.019	-.220	13.820	13.830	748.4	-2195.7	24.3	2195.8
0.020	-.205	14.568	14.579	751.0	-2184.4	-29.4	2184.6
0.021	-.191	15.319	15.331	753.7	-2174.2	-86.5	2175.9
0.022	-.175	16.073	16.085	756.3	-2165.2	-146.9	2170.2
0.023	-.158	16.829	16.842	758.8	-2157.9	-210.8	2168.1
0.024	-.141	17.588	17.602	761.2	-2152.4	-278.2	2170.3
0.025	-.123	18.349	18.364	763.5	-2149.1	-349.0	2177.2
0.026	-.104	19.113	19.128	765.6	-2148.3	-423.2	2189.6
0.027	-.085	19.878	19.894	767.6	-2150.4	-500.8	2207.9
0.028	-.065	20.646	20.661	769.3	-2155.8	-581.7	2232.9
0.029	-.044	21.415	21.431	770.7	-2164.8	-665.8	2264.9
0.030	-.022	22.186	22.202	771.8	-2177.9	-753.0	2304.4
0.031	0.001	22.958	22.973	772.5	-2195.4	-843.1	2351.7
0.032	0.024	23.730	23.746	772.9	-2217.8	-935.9	2407.1
0.033	0.048	24.503	24.518	772.9	-2245.4	-1031.1	2470.9
0.034	0.073	25.276	25.290	772.3	-2278.8	-1128.5	2543.0
0.035	0.099	26.048	26.062	771.3	-2318.4	-1227.7	2623.4
0.036	0.125	26.820	26.832	769.7	-2364.4	-1328.4	2712.0
0.037	0.152	27.589	27.600	767.5	-2417.4	-1430.0	2808.7
0.038	0.180	28.357	28.366	764.6	-2477.8	-1532.1	2913.2
0.039	0.209	29.122	29.129	761.0	-2545.8	-1634.2	3025.1
0.040	0.239	29.883	29.887	756.7	-2621.8	-1735.6	3144.2
0.041	0.269	30.639	30.642	751.6	-2706.1	-1835.6	3269.9

0.042	0.300	31.391	31.390	745.7	-2798.8	-1933.7	3401.8
0.043	0.332	32.137	32.133	738.8	-2900.3	-2028.9	3539.5
0.044	0.364	32.875	32.868	731.0	-3010.6	-2120.5	3682.4

APPENDIX 2

```
program scourt2bis

      integer n
      real mh,mt,Icgh,rcgh,rcgt,Vcd,Foxt(45),Fsb(45),F1xh(45),
     +F1yh(45),thetdd(45),thetad(45),thetd(45),theta(45),A,B,
     +t(45),g,C(45),D(45),E(45),H(45),Foy(45),F1xt(45),F1yt(45),
     +Fl,ml,Fox(45),r1,r2,J(45),K(45),L(45),Foyt(45),phi,Fo(45),
     +Fsbx(45),Fsby(45),F1t(45)
C  mh: mass of the head, in slugs
C  mt: mass of the torso, in slugs
C  Icgh: Moment of inertia of the head about the center of gravity of
C        the head, in slug-feet squared
C  rcgh: distance from the neck to the center of gravity of the head,
C         in feet
C  rcgt: distance from the waist to the center of gravity of the torso,
C        in feet.
C  Vcd: rate of deceleration of the vehicle, in feet per second squared
C  Foxt: x component of the force of the lap belt on the torso, in
C          pounds
C  Fsb: force of the shoulder belt on the chest, in pounds
C  F1xh: x component of the force of the torso on the head, in pounds
C  F1yh: y component of the force of the torso on the head, in pounds
C  thetdd: angular acceleration of the head, in radians per second
C          squared
C  thetad: angular velocity of the head, in radians per second
C  thetd: closed form solution for the angular velocity of the head,
C         in radians per second
C  theta: angle of the head with the vertical, in radians
C  t: time in seconds, from the onset of collision
C  g: acceleration due to gravity, in feet per second squared
C  Foy: y component of the force of the lap belt on the wearer, in
C       pounds
C  F1xt: x component of the force of the head on the torso, in pounds
C  F1yt: y component of the force of the head on the torso, in pounds
C  F1t: force of the head on the torso, in pounds
C  Fl: force of the lap belt on the lower body, in pounds
C  ml: mass of the lower body, in slugs
C  Fox: x component of the force of the lap belt on the wearer, in
pounds
C  r1: distance from the waist to mid neck, in feet
C  Foyt: y component of the force of the lap belt on the torso, in
C        pounds
C  Fo: force of the lap belt on the wearer, in pounds
C  phi: angle the torso makes with the vertical, in radians
      mh=0.287
      mt=1.7
      ml=1.64
      rcgt=0.68
      rcgh=0.44
      r1=1.47
```

```
      r2=rcgt-r1
      Icgh=0.0173
      phi=-0.35
      dt=0.001
      theta(1)=0
      thetad(1)=0
      t(1)=0
      Vcd=968.0
      g=32.2
      A=g*mh*rcgh/(Icgh+mh*(rcgh**2))
      B=A*Vcd/g
      thetdd(1)=A*SIN(theta(1))+B*COS(theta(1))
      DO 10 n=2,45
      thetad(n)=thetad(n-1)+dt*thetdd(n-1)
      theta(n)=theta(n-1)+dt*(thetad(n-1)+thetad(n))/2.0
      thetdd(n)=A*SIN(theta(n))+B*COS(theta(n))
      thetd(n)=SQRT(2.0*A*COS(theta(1))-2.0*A*COS(theta(n))+
     +2.0*B*SIN(theta(n))-2.0*B*SIN(theta(1)))
      t(n)=t(n-1)+dt
      C(n)=thetdd(n)*rcgh*COS(theta(n))
      D(n)=rcgh*(thetad(n)**2)*SIN(theta(n))
      F1xh(n)=mh*(C(n)-D(n)-Vcd)
      E(n)=g-rcgh*thetdd(n)*SIN(theta(n))
      H(n)=rcgh*(thetad(n)**2)*COS(theta(n))
      F1yh(n)=mh*(E(n)-H(n))
      F1yt(n)=-F1yh(n)
      F1xt(n)=-F1xh(n)
      F1t(n)=SQRT((F1xt(n)**2)+(F1yt(n)**2))
      Fl=ml*Vcd
      J(n)=r2*SIN(phi)*F1yt(n)-r2*COS(phi)*F1xt(n)
      K(n)=rcgt*SIN(phi)*COS(phi)*(F1yt(n)-mt*g)
      L(n)=rcgt*(COS(phi)**2)*(F1xt(n)+mt*Vcd)
      Fsbx(n)=(K(n)-J(n)-L(n))/rcgt
      Fsby(n)=-Fsbx(n)*TAN(phi)
      Foxt(n)=-mt*Vcd-F1xt(n)-Fsbx(n)
      Foyt(n)=mt*g-F1yt(n)-Fsby(n)
      Fsb(n)=SQRT(Fsbx(n)**2+Fsby(n)**2)
      Fox(n)=Foxt(n)-Fl
      Fo(n)=SQRT((Fox(n)**2)+(Foyt(n)**2))
   10 CONTINUE
      PRINT 20
   20 FORMAT('',T4,'t',T8,'theta',T16,'thetad',T25,'thetdd',T35,
     +'Fox',T45,'F1xt',T54,'F1yt',T62,'Fsb')
      DO 30 n=2,45
      PRINT 25, t(n),theta(n),thetad(n),thetdd(n),Fox(n),
     +F1xt(n),F1yt(n),Fsb(n)
   25 FORMAT(F5.3,2X,F5.3,2X,F7.3,2X,F7.1,2X,F7.1,2X,F7.1,2X,F7.1,
     +2X,F7.1)
   30 CONTINUE
      PRINT 35
   35 FORMAT(//,T4,'t',T11,'Fsbx',T20,'Fsby',T29,'Foxt',T38,'Foyt',
     +T47,'Fo',T56,'F1t')
      DO 45 n=2,45
      PRINT 40, t(n),Fsbx(n),Fsby(n),Foxt(n),Foyt(n),Fo(n),F1t(n)
   40 FORMAT(F5.3,2X,F7.1,2X,F7.1,2X,F7.1,2X,F7.1,2X,F7.1,2X,F7.1)
   45 CONTINUE
      END
```

Table 2

t	theta	thetad	thetdd	Fox	F1xt	F1yt	Fsb
0.001	0.001	1.678	1677.7	-1740.7	66.0	-8.7	1659.0
0.002	0.003	3.355	1677.8	-1739.5	65.9	-7.1	1660.2
0.003	0.008	5.033	1678.0	-1737.6	65.9	-4.4	1662.2
0.004	0.013	6.711	1678.2	-1734.9	66.0	-0.7	1665.2
0.005	0.021	8.389	1678.5	-1731.3	66.1	4.1	1669.1
0.006	0.030	10.068	1678.6	-1726.9	66.3	10.0	1674.1
0.007	0.041	11.746	1678.5	-1721.5	66.7	16.9	1680.3
0.008	0.054	13.425	1678.2	-1715.1	67.4	24.9	1687.8
0.009	0.068	15.103	1677.6	-1707.6	68.4	33.9	1696.8
0.010	0.084	16.781	1676.4	-1699.0	69.8	43.9	1707.5
0.011	0.102	18.457	1674.7	-1689.1	71.8	55.0	1720.1
0.012	0.121	20.132	1672.1	-1677.9	74.4	67.0	1734.7
0.013	0.142	21.804	1668.7	-1665.4	77.7	80.0	1751.6
0.014	0.164	23.473	1664.2	-1651.3	81.9	93.8	1771.1
0.015	0.189	25.137	1658.3	-1635.7	87.1	108.4	1793.2
0.016	0.215	26.795	1651.0	-1618.5	93.4	123.8	1818.3
0.017	0.242	28.446	1642.0	-1599.6	101.0	139.7	1846.5
0.018	0.272	30.088	1631.1	-1579.0	110.1	156.1	1878.1
0.019	0.302	31.719	1618.1	-1556.6	120.6	172.9	1913.1
0.020	0.335	33.338	1602.7	-1532.5	132.8	189.8	1951.7
0.021	0.369	34.940	1584.8	-1506.7	146.8	206.7	1994.1
0.022	0.405	36.525	1564.0	-1479.3	162.6	223.4	2040.1
0.023	0.442	38.089	1540.2	-1450.3	180.4	239.6	2089.9
0.024	0.481	39.629	1513.1	-1420.0	200.2	255.0	2143.3
0.025	0.521	41.142	1482.5	-1388.4	222.0	269.4	2200.0
0.026	0.563	42.625	1448.2	-1355.9	245.7	282.4	2259.9
0.027	0.607	44.073	1410.1	-1322.7	271.4	293.8	2322.6
0.028	0.651	45.483	1367.9	-1289.2	298.8	303.2	2387.6
0.029	0.698	46.851	1321.6	-1255.6	328.0	310.4	2454.3
0.030	0.745	48.173	1270.9	-1222.6	358.6	315.0	2522.0
0.031	0.794	49.444	1215.9	-1190.4	390.3	316.7	2590.1
0.032	0.844	50.659	1156.5	-1159.7	423.0	315.2	2657.5
0.033	0.895	51.816	1092.7	-1130.8	456.1	310.5	2723.5
0.034	0.948	52.909	1024.5	-1104.5	489.4	302.1	2787.0
0.035	1.001	53.933	952.0	-1081.1	522.3	290.1	2846.9
0.036	1.055	54.885	875.4	-1061.1	554.3	274.4	2902.2
0.037	1.111	55.760	794.9	-1045.2	585.1	255.0	2952.0
0.038	1.167	56.555	710.6	-1033.6	614.0	232.0	2995.0
0.039	1.224	57.266	623.0	-1026.8	640.5	205.6	3030.5
0.040	1.281	57.889	532.3	-1025.1	664.2	176.0	3057.6
0.041	1.340	58.421	438.8	-1028.6	684.6	143.5	3075.6
0.042	1.398	58.860	343.1	-1037.4	701.4	108.6	3084.0
0.043	1.457	59.203	245.6	-1051.6	714.1	71.7	3082.4
0.044	1.517	59.449	146.7	-1071.0	722.4	33.5	3070.7

APPENDIX 3

```
program scourt3
      integer n
      real xr,rcg,dtheta,theta1,theta2,theta(30),delt,thetad(30),mt,ml,
     +Icgt,Vcd,g,Fsbx(30),Fsby(30),Foxt(30),Foyt(30),Fox(30),A(30),Fl,
     +B(30),C(30),t(30),dt,thetad2,Fo(30),Fsb(30)
C  xr: the depth to which the chest is crushed, in feet
C  rcg: the distance from the waist to the center of gravity of the
C      torso, in feet
C  dtheta: change in the angle of the torso, in radians
C  theta1: initial angle of the torso, in radians
C  theta2: final angle of the torso, in radians
C  theta: angle of the torso, in radians
C  delt: time interval to go from theta1 to theta2, in seconds
C  thetad: angular velocity, in radians per second
C  mt: mass of the torso, in slugs
C  m1: mass of the body below the waist, in slugs
C  Icgt: moment of inertia of the torso about the center of gravity of
C       the torso, in slug-feet squared
C  Vcd: rate of change of the velocity of the car, in feet per second
C      squared
C  g: acceleration due to gravity, in feet per second squared
C  Fsbx: x component of the force of the shoulder belt on the torso,
C       in pounds
C  Fsby: y component of the force of the shoulder belt on the torso,
C       in pounds
C  Foxt: x component of the force of the lap belt on the torso, in
C       in pounds
C  Foyt: y component of the force of the lap belt on the torso, in
C        pounds
C  Fox:  x component of the total force of the lap belt on the body,
C        in pounds
C  Fl: force of the lap belt on the body below the waist, in pounds
C: t: time from the onset of collision, in seconds
C: dt: time interval per step used in the numerical integration, in
C      seconds
C  thetad2: final angular velocity of the torso (=0)
C: Fo: total force of the lap belt on the body, in pounds
C: Fsb: total force of the shoulder belt on the chest, in pounds

      xr=0.167
      rcg=0.68
      dtheta=xr/rcg
      theta(1)=-0.19
      theta2=theta1+dtheta
      thetad(1)=16.21
      thetad2=0
      delt=2.0*dtheta/thetad(1)
      thetdd=(thetd2-thetad(1))/delt
      mt=1.7
      ml=1.64
      t(1)=0.021
      Vcd=-968.0
      Icgt=0.284
      g=32.2
```

```
     dt=0.001
     Fl=ml*Vcd
     DO 10 n=2,30
     thetad(n)=thetad(n-1)+thetdd*dt
     theta(n)=theta(n-1)+thetad(n-1)*dt
     A(n)=mt*(Vcd*rcg*COS(theta(n))+(rcg**2)*thetdd-
    +g*rcg*SIN(theta(n)))
     Fsbx(n)=(Icgt*thetdd+A(n))/(2.0*rcg*COS(theta(n)))
     Fsby(n)=-Fsbx(n)*TAN(theta(n))
     Fsb(n)=SQRT(Fsbx(n)**2+Fsby(n)**2)
     B(n)=Vcd+rcg*thetdd*COS(theta(n))-rcg*(thetad(n)**2)*
    +SIN(theta(n))
     FoxT(n)=-Fsbx(n)+mt*B(n)
     C(n)=g-rcg*thetdd*SIN(theta(n))-rcg*(thetad(n)**2)*
    +COS(theta(n))
     Foyt(n)=-Fsby(n)+mt*C(n)
     Fox(n)=Foxt(n)+Fl
     Fo(n)=SQRT(Fox(n)**2+Foyt(n)**2)
     t(n)=t(n-1)+dt
  10 CONTINUE
     PRINT 20
  20 FORMAT(//,T4,'t',T10,'Fsbx',T19,'Fsby',T29,'Foxt',T38,'Foyt',
    +T46,'Fox',T53,'theta',T62,'Fsb')
     DO 30 n=2,30
     PRINT 25, t(n),Fsbx(n),Fsby(n),Foxt(n),Foyt(n),Fox(n),theta(n),
    +Fsb(n)
  25 FORMAT(F5.3,2X,F7.1,2X,F7.1,2X,F7.1,2X,F7.1,2X,F7.1,2X,F5.3,
    +2X,F7.1)
  30 CONTINUE
     PRINT 35
  35 FORMAT(//,T3,'Fo')
     DO 50 n=2,30
     PRINT 40, Fo(n)
  40 FORMAT(F7.1)
  50 CONTINUE
     END
```

Table 3

t	theta	thetad	thetdd	Fox	F1xt	F1yt	Fsb
0.001	0.001	1.678	1677.7	-1740.7	66.0	-8.7	1659.0
0.002	0.003	3.355	1677.8	-1739.5	65.9	-7.1	1660.2
0.003	0.008	5.033	1678.0	-1737.6	65.9	-4.4	1662.2
0.004	0.013	6.711	1678.2	-1734.9	66.0	-0.7	1665.2
0.005	0.021	8.389	1678.5	-1731.3	66.1	4.1	1669.1
0.006	0.030	10.068	1678.6	-1726.9	66.3	10.0	1674.1
0.007	0.041	11.746	1678.5	-1721.5	66.7	16.9	1680.3
0.008	0.054	13.425	1678.2	-1715.1	67.4	24.9	1687.8
0.009	0.068	15.103	1677.6	-1707.6	68.4	33.9	1696.8
0.010	0.084	16.781	1676.4	-1699.0	69.8	43.9	1707.5
0.011	0.102	18.457	1674.7	-1689.1	71.8	55.0	1720.1
0.012	0.121	20.132	1672.1	-1677.9	74.4	67.0	1734.7
0.013	0.142	21.804	1668.7	-1665.4	77.7	80.0	1751.6
0.014	0.164	23.473	1664.2	-1651.3	81.9	93.8	1771.1
0.015	0.189	25.137	1658.3	-1635.7	87.1	108.4	1793.2
0.016	0.215	26.795	1651.0	-1618.5	93.4	123.8	1818.3
0.017	0.242	28.446	1642.0	-1599.6	101.0	139.7	1846.5
0.018	0.272	30.088	1631.1	-1579.0	110.1	156.1	1878.1
0.019	0.302	31.719	1618.1	-1556.6	120.6	172.9	1913.1
0.020	0.335	33.338	1602.7	-1532.5	132.8	189.8	1951.7
0.021	0.369	34.940	1584.8	-1506.7	146.8	206.7	1994.1
0.022	0.405	36.525	1564.0	-1479.3	162.6	223.4	2040.1
0.023	0.442	38.089	1540.2	-1450.3	180.4	239.6	2089.9
0.024	0.481	39.629	1513.1	-1420.0	200.2	255.0	2143.3
0.025	0.521	41.142	1482.5	-1388.4	222.0	269.4	2200.0
0.026	0.563	42.625	1448.2	-1355.9	245.7	282.4	2259.9
0.027	0.607	44.073	1410.1	-1322.7	271.4	293.8	2322.6
0.028	0.651	45.483	1367.9	-1289.2	298.8	303.2	2387.6
0.029	0.698	46.851	1321.6	-1255.6	328.0	310.4	2454.3
0.030	0.745	48.173	1270.9	-1222.6	358.6	315.0	2522.0
0.031	0.794	49.444	1215.9	-1190.4	390.3	316.7	2590.1
0.032	0.844	50.659	1156.5	-1159.7	423.0	315.2	2657.5
0.033	0.895	51.816	1092.7	-1130.8	456.1	310.5	2723.5
0.034	0.948	52.909	1024.5	-1104.5	489.4	302.1	2787.0
0.035	1.001	53.933	952.0	-1081.1	522.3	290.1	2846.9
0.036	1.055	54.885	875.4	-1061.1	554.3	274.4	2902.2
0.037	1.111	55.760	794.9	-1045.2	585.1	255.0	2952.0
0.038	1.167	56.555	710.6	-1033.6	614.0	232.0	2995.0
0.039	1.224	57.266	623.0	-1026.8	640.5	205.6	3030.5
0.040	1.281	57.889	532.3	-1025.1	664.2	176.0	3057.6
0.041	1.340	58.421	438.8	-1028.6	684.6	143.5	3075.6
0.042	1.398	58.860	343.1	-1037.4	701.4	108.6	3084.0
0.043	1.457	59.203	245.6	-1051.6	714.1	71.7	3082.4
0.044	1.517	59.449	146.7	-1071.0	722.4	33.5	3070.7