ON THE EFFECT OF SEATBELTS IN A REAL SCHOOL BUS ACCIDENT

A Safety Choice Coalition Report

The Safety Choice Coalition
P.O. Box 2591
Boise, ID 83701
208-429-1509

School buses are among the safest forms of transportation known to man (1). Accidents involving fatalities on school buses are extremely rare (2). Nevertheless, some members of congress have put pressure on NHTSA to get states to put seatbelts on school buses. As a result, numerous bills have been introduced in state legislatures to require that school buses be equipped with seatbelts. So far, few of these bills have passed.

The present paper is based on a video posted on youtube which shows a fatal head-on collision between a car and a school bus (3). All the fatalities occurred in the car. The description accompanying the video states that “no one on the bus was seriously hurt”. In the video we see the children calmly getting off the bus after the accident, and none appear to be injured. The car was badly crushed and all those in the car were killed. No damage to the school bus is visible in the video. This accident took place in Arizona where school buses are not equipped with seatbelts.

The collision shown in the video was not, strictly speaking, a head-on collision in the sense that the car and the bus were not perfectly aligned on the same axis. However, this is about as close to a textbook head-on collision as one is likely to get in real life. Because the bus and the car were not perfectly aligned, we see the bus pushing the car, or what is left of it, to one side after the collision. This enables us to see the extent to which the car was crushed by the collision. In order to obtain a more exact estimate we have extracted a still from the video, showing the extent of the crush:

Figure 1

The bus does not appear to have been crushed at all. This is to be expected from the way cars and school buses are made. School buses have chassis, modern cars do not. In addition to estimating the extent to which the car was crushed from the picture, we can estimate the speed of the two vehicles prior to collision from the video. We thus have enough information to calculate the rates of deceleration of the bus and the car, and hence we can calculate the force which the passengers on the bus would have experienced had they been wearing seatbelts.

The law of the conservation of energy gives us

$$m_c \frac{V_{c1}^2}{2} + m_b \frac{V_{b1}^2}{2} = (m_b + m_c)\frac{V_{b2}^2}{2} + W \tag{1}$$

where m_c = mass of the car
m_b = mass of the bus
V_{c1} = velocity of the car at impact
V_{b1} = velocity of the bus at impact
V_{b2} = velocity of the bus and car after impact
W = work of compression on the car

The forces on the part of the car being crushed are shown in Figure 2.

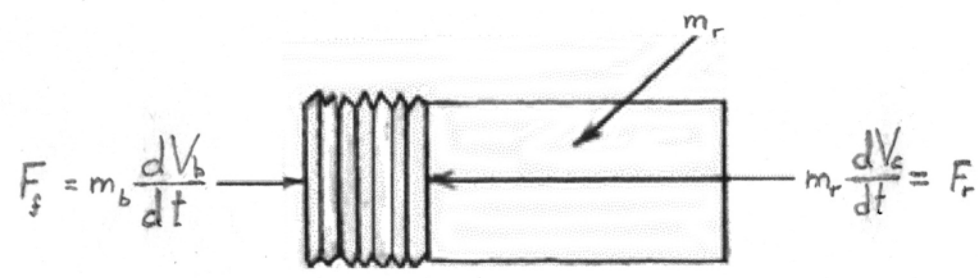

Figure 2

The crush propagates back from the front end of the car. The force on the front end of the crush zone is $F_f = m_b dV_b/dt$ and the force on the back end of the crush zone is $F_r = m_r dV_c/dt$, where m_r is the mass of the uncrushed part of the car. The force $F_f - F_r$ gives us the rate of change of the velocity of the crush zone, integrated over the zone, while the work of crush, W, is the integral of $F_r dx$ from x = 0 to $x = x_f$, where x_f is the amount of crush (i.e., the original length of the part of the car which is crushed). Notice that the work of crush is independent of the coordinate system and depends only on the internal properties of the structure.

We may approximate m_r by the equation

$$m_r = m_c - \frac{x}{l} m_c \qquad (2)$$

where l is the length of the car and x the distance crushed. The work is then

$$W = \int_0^{x_f} m_r \frac{dV_c}{dt} dx = \int_0^{x_f} m_c (1 - \frac{x}{l}) a_c dx \qquad (3)$$

where $a_c = \frac{dV_c}{dt}$ and x_f is the total length of the of the part of the car which is crushed. We take $a_c = \frac{V_{b2} - V_{c1}}{t_2}$ bearing in mind that V_{c1} is negative and a_c is positive. t_2 is the time from the onset of collision to the time the bus and the car have reached the same velocity and the crush has been completed. t_2 may be found from the equation

$$\frac{1}{t_2} = \frac{\overline{V_b} - \overline{V_c}}{x_f} \qquad (4)$$

where $\overline{V_b}$ and $\overline{V_c}$ are the average velocities of the bus and car during the collision taken positive to the right, so $\overline{V_c}$ will be negative. We take

$$\overline{V_b} = \frac{V_{b1} + V_{b2}}{2} \quad \text{and} \quad \overline{V_c} = \frac{V_{b2} + V_{c1}}{2} \tag{5}$$

Substituting in (4),

$$\frac{1}{t_2} = \frac{1}{x_f}\left[\frac{V_{b1} + V_{b2}}{2} - \frac{V_{b2} + V_{c1}}{2}\right] \tag{6}$$

$$\frac{1}{t_2} = \frac{1}{x_f}\left[\frac{V_{b1} - V_{c1}}{2}\right] \tag{7}$$

where V_{c1} is negative. (For an alternative approach to get the same result, see Appendix 1).

The deceleration of the car, a_c , will be given by

$$a_c = \frac{V_{b2} - V_{c1}}{t_2} \tag{8}$$

The work W is thus given by

$$W = \int_0^{x_f} m_r \frac{dV_c}{dt} dx = \int_0^{x_f} m_r a_c dx \tag{9}$$

$$m_r = m_c - \frac{x}{l} m_c = m_c\left(1 - \frac{x}{l}\right) \tag{10}$$

$$\therefore W = \int_0^{x_f} m_c\left(1 - \frac{x}{l}\right) a_c dx = m_c a_c x_f - \frac{m_c a_c}{l}\frac{x_f^2}{2} \tag{11}$$

or

$$W = m_c a_c x_f\left(1 - \frac{1}{2}\frac{x_f}{l}\right) = m_c \frac{V_{b2} - V_{c1}}{t_2} x_f\left(1 - \frac{1}{2}\frac{x_f}{l}\right) \tag{12}$$

from equation (8). Substituting in equation (1),

$$m_b \frac{V_{b1}^2}{2} + m_c \frac{V_{c1}^2}{2} = \left(m_b + m_c\right)\frac{V_{b2}^2}{2} + m_c \frac{V_{b2} - V_{c1}}{t_2} x_f \left(1 - \frac{1}{2}\frac{x_f}{l}\right) \tag{13}$$

This is a quadratic equation for V_{b2} with the solution

$$V_{b2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{14}$$

where
$$A = \frac{m_b + m_c}{2}, \quad B = \frac{m_c}{t_2} x_f \left(1 - \frac{1}{2}\frac{x_f}{l}\right) \tag{15}$$

and
$$C = -\frac{m_c}{t_2} V_{c1} x_f \left(1 - \frac{1}{2}\frac{x_f}{l}\right) - \left(m_b \frac{V_{b1}^2}{2} + m_c \frac{V_{c1}^2}{2}\right) \tag{16}$$

In order to solve the equation for arbitrary parameters we have written a short computer program (Appendix 2). The minus sign gives us and extraneous root, so the correct form of the solution is

$$V_{b2} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \tag{17}$$

From the video (3) we estimate V_{b1} = 37 fps, V_{c1} = 52 fps. From the still (Figure 1) we estimate x_f at 6 feet. We take the initial length of the car as 16 feet and m_c = 124 slugs (4,000 pounds). m_b is taken as 667 slugs (21,500 pounds), from reference (4).

Running a solution with these numbers we find V_{b2} = 26.3 fps, t_2 = 0.13 seconds, a_b = 79.5 ft/sec^2 and a_c = 580 ft/sec^2. A deceleration of 79.5 ft/sec^2 is equivalent to 2.47g. An 80 pound child tied to a seat by a seatbelt would thus experience a force of roughly 197 pounds from the seatbelt (5). The exact force and force distribution would depend on the type of seatbelt but, as we have seen from our analysis of the effect of a shoulder belt (6), the force would be of the same order of magnitude regardless of the type of belt. Thus, if the children had been wearing seatbelts, they would have been severely injured. Because they were not wearing seatbelts they emerged with only minor injuries, or no injuries. The description accompanying the video only says “nobody on the bus was seriously hurt”. None of the children getting off the bus, as seen on the video, appear to be injured at all. The occupants of the front seat of the car would, of course, have been crushed to death. With a deceleration of 580 ft/sec^2 , anyone in the car wearing a seatbelt would, if not crushed to death, have been killed by the seatbelt.

References:

1. www.safetychoice.org/Documents/school_bus.htm
2. ftp://ftp.nhtsa.dot.gov/fars/
3. www.youtube.com/watch?v=_6ONThNwmK8&feature=related
4. www.fldoe.org/board/meetings/2008_6_17/2008Bus.pdf
5. www.safetychoice.org/Documents/solution.htm
6. www.sasfetychoice.org/Documents/scourt.pdf

APPENDIX 1

Using absolute values we get, from reference (5),

$$x_f = V_{b1}t_2 - a_b \frac{t_2^2}{2} + V_{c1}t_2 - a_c \frac{t_2^2}{2} \quad (1a)$$

Taking $a_c = \frac{V_{c1} + V_{b2}}{t_2}$ and $a_b = \frac{V_{b1} - V_{b2}}{t_2}$ (2a)

Substituting equations (2a) into (1a), we get

$$x_f = V_{b1}t_2 - \left(\frac{V_{b1} - V_{b2}}{t_2}\right)\frac{t_2^2}{2} + V_{c1}t_2 - \left(\frac{V_{c1} + V_{b2}}{t_2}\right) \quad (3a)$$

$$= \frac{V_{b1} + V_{c1}}{2} t_2 \quad (4a)$$

Thus, $\frac{1}{t_2} = \frac{V_{b1} + V_{c1}}{2x_f}$ (5a)

which is the same as equation (7), bearing in mind that in equation (7) V_{c1} is negative.

APPENDIX 2

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      program bus
      real Vb1,Vc1,W,mc,mb,mr,l,xf,ac,ab,t2,A,B,C1,C2,C,Vb2,Vb2a,Vb2b
C   Vb1 = initial velocity of bus, ft/sec
C   Vc1 = initial velocity of car, ft/sec
C   W = work of compression, ft-lbs
C   mc = mass of car, slugs
C   mb = mass of bus, slugs
C   mr = uncrushed mass of car, slugs
C   l = original length of car, feet
C   xf = length of part of car which is crushed, feet
C   t2 = time from onset of collision to final velocity, ft/sec
C   ac = rate of deceleration of car, ft/second squared
C   ab = rate of deceleration of bus, ft/second squared
      Vb1=37
      Vc1=52
      xf=6
      mc=124
      mb=667
      l=16
      t2=2.0*xf/(Vb1+Vc1)
      A=(mb+mc)/2.0
      B=mc*xf*(1.0-xf/(2.0*l))/t2
      C1=B*Vc1
      C2=mb*(Vb1**2)/2.0+mc*(Vc1**2)/2.0
      C=C1-C2
      Vb2=(-B+SQRT(B**2-4.0*A*C))/(2.0*A)
      ab=(Vb1-Vb2)/t2
      ac=(Vc1+Vb2)/t2
      PRINT 10,Vb2,t2,ab,ac
   10 FORMAT('','Vb2=',F5.1,4X,'t2=',F4.2,4X,'ab=',F6.1,4X,'ac=',F6.1)
      END
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