

SOLUTION TO PUZZLE NUMBER 2

All data are taken from Yamaguchi (1) except the weight and dimensions of the child, which are taken from CDC (2), and the crush, which is estimated from photographs of actual rollovers (refs 3, 4).

Referring to Figure 1, the original figure is from Yamaguchi (1). The ground level after impact and the occupant figure on the outside of the rotation have been added. The occupant figure given by Yamaguchi is for an adult but the center of gravity of the child has been taken in the same place, due to the elevation provided by the booster seat. The dimensions of the occupants have been adjusted in the calculations however, to correspond to those of a four year old child.

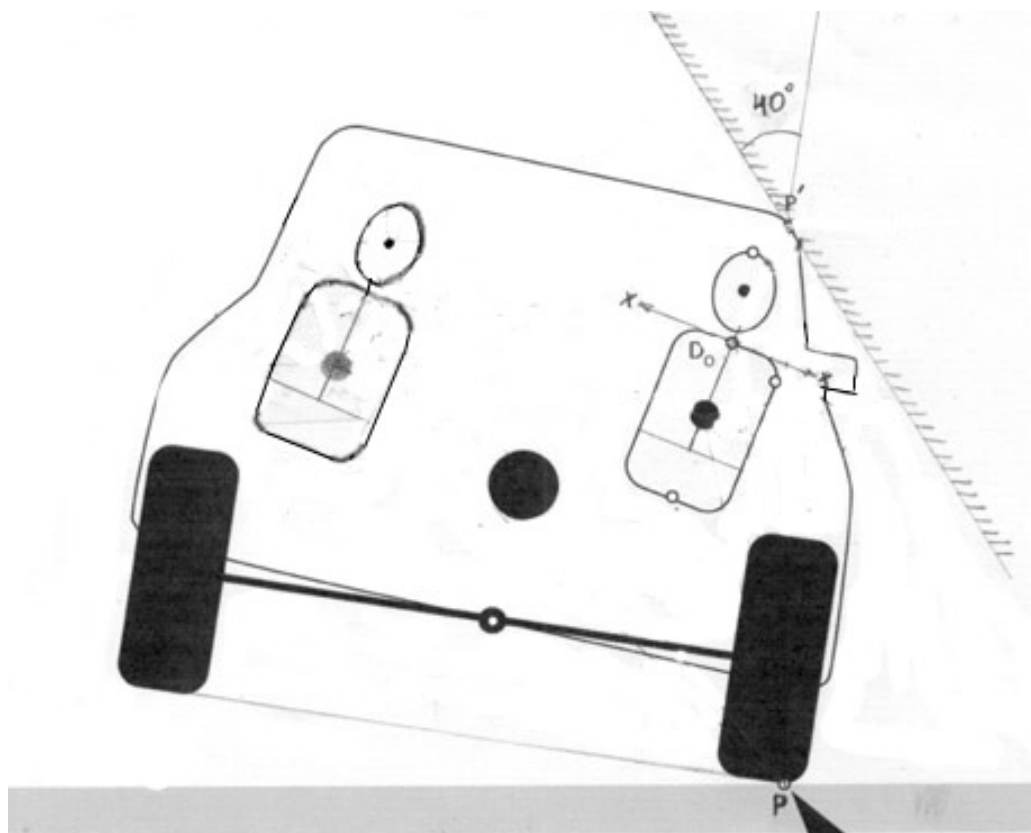


Figure 1

The vehicle considered by Yamaguchi is somewhat unusual in that the width of the roof is considerably less than that of the base. As a result, the vehicle rotates 130° before impact. According to the FARS data base, most fatal rollovers involve SUV's or vans, where the roof width is closer to the width of the base. In those cases the rotation before impact would be closer to 90° and the force components into the neck of the child would be greater, while those down on the shoulder would be less.

For those not familiar with rollovers, when the vertical component of the centrifugal force due to the rotation exceeds the weight of the vehicle, the vehicle will become airborne. This is known as a "vault". The reason this is possible is that the centrifugal force of the roll, not to be confused with the centrifugal force due to the path of the vehicle, may become greater than the weight of the vehicle while the vertical component of that force will still be less. As the angle of rotation increases, the vertical component increases, reaching a maximum when the angle is 90° . At some point, therefore, the vertical component of the centrifugal force may exceed the weight of the vehicle.

In the present case, the vehicle becomes airborne at 90° (Figures 2, 3 and 4). The vehicle has only rotated 50° at this

point, but the center of gravity is directly above the axis of rotation. Thus, the centrifugal force is vertical; that is, perpendicular to the ground, and hence the vertical component of the force is equal to the total force. Once the vehicle becomes airborne, the axis of rotation shifts from the bottom of the tires on the right side of the vehicle (in the present case) to the center of gravity. The horizontal component of the velocity of the center of gravity is conserved, while the vertical component experiences a downward acceleration of 32.2 feet per second squared. In addition, the angular momentum is conserved, so that the rate of rotation about the cg remains constant until the roof edge strikes the ground.

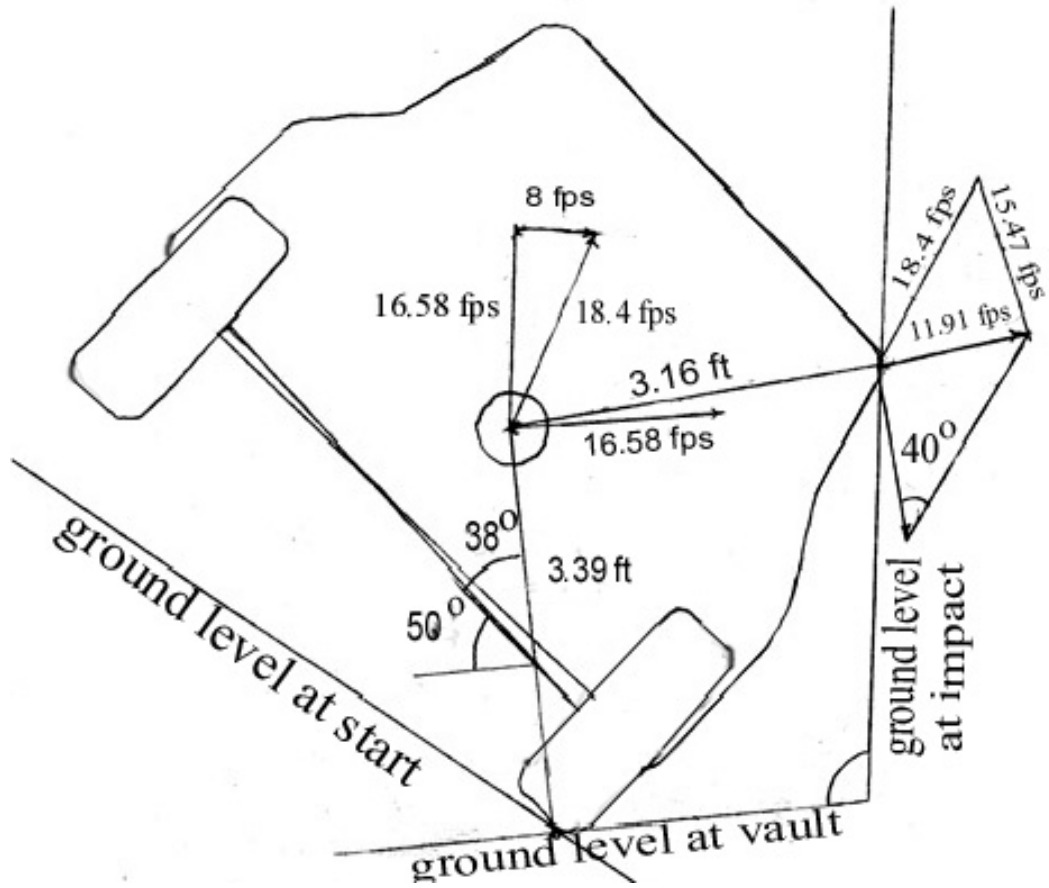


Figure 2

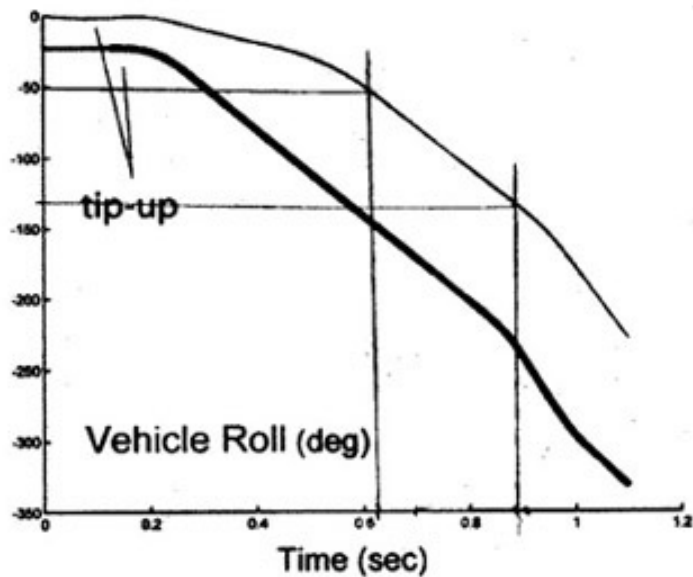


Figure 3

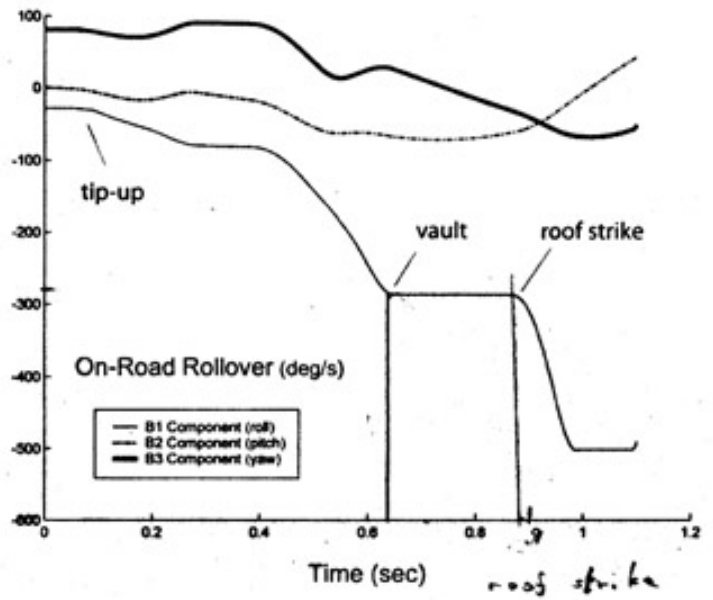


Figure 4

The velocity vector at each point in the vehicle will be the vector sum of the velocity of the center of gravity plus the velocity of the point relative to the center of gravity. Thus the velocity vector of the center of gravity of the child, whether sitting on the inside or the outside of the rotation, may be determined (Figure 5). When the roof edge strikes the ground, the component of the velocity vector in the direction of the point of impact must go to zero, since no more motion in that direction is possible. There will, therefore, be a force on the child in that direction. The magnitude that force will depend on how fast that component of the velocity vector goes to zero, in accordance with Newton's second law of motion. That, in turn, depends on the extent to which the vehicle is stove in at the point of impact. If there were no crush, and the child was firmly connected to the vehicle, the force would be infinite.

We may estimate the degree of crush on impact from photographs of actual rollover accidents, as shown in our picture gallery, elsewhere on this web-site, as well as the photographs of Larsen (4). It is important to bear in mind that the crush on impact is not the final crush, since the roof will be stove in as the vehicle continues to rotate. From an assumed impact crush we may calculate the resulting force on the body shell, which gives an additional check on the validity of the assumption. Based on these considerations, we have taken the impact crush as 6 inches in the present case. After the impact crush, the impact point become the new center of rotation. That is, the axis of rotation shifts from the center of gravity to the impact point, P'.

Since the width of the vehicle at the base is not given, we estimate it to be 6 feet, from the cross-section shown in Figure 1. From the figure it follows that the distance from the initial axis of rotation to the center of gravity of the vehicle is 3.39 feet. From figure 4 we find that the angular velocity at impact is 4.89 radians per second, giving us a linear velocity of 16.58 fps for the center of gravity parallel to the ground. The vehicle is airborne for 0.25 seconds (figure 4) so that for a downward acceleration of 32 feet per second squared, the downward velocity component of the cg is 8 feet per second at impact. The vector sum gives a velocity of 18.4 feet per second of the cg at impact. To get the velocity of the impact point we have to add the velocity due to the rotation about the cg. From Figure 1, the distance from the cg to the impact point is 3.16 feet and hence the rotational velocity is $3.16 \times 4.89 = 15.47$ fps. Taking the vector sum of the translational and rotational velocities we find that the velocity of the impact point at impact is 11.91 fps. (figure 2). For a crush of 6 inches, the time it takes for the velocity at P' to go to zero is 0.084 seconds:

Let s be the displacement of the point P', V the velocity, t the time and a the deceleration. Then

$$(1) \frac{d^2s}{dt^2} = a$$

$$(2) \frac{ds}{dt} = at + c_1 = V$$

$$(3) s = at^2 / 2 + c_1t + c_2$$

The boundary conditions are, at $t = 0$, $s = 0$ and $V = 11.91$ fps

At $s = 0.5$ feet, $V = 0$ and $t = t_2$, the time it takes for the point P' to come to rest.

Hence, $c_1 = 11.91$ and $c_2 = 0$. Thus,

$$at_2 + 11.91 = 0 \text{ or } a = -11.91/t_2$$

$$\text{Substituting in equation (3), } \frac{1}{2} = -\frac{11.91}{2}t_2 + 11.91t_2 = \frac{11.91}{2}t_2$$

$$\text{Thus, } t_2 = 1/11.91 = .084 \text{ seconds.}$$

The easiest way to calculate the velocity vectors for the children in the booster seats is graphically. This is certainly not precise but it is accurate enough for our purposes. Using this method (figure5) we find that the velocity of the cg of the outside child at impact is 26.77 fps and the vector change in the velocity is 12.57 fps at an angle of 48.5 degrees to vertical axis of the child.

(Yamaguchi shows the occupants at an angle. This would be the position of the occupants when their necks abut the shoulder belt, since the belt may not be initially directly against the neck). The time to stop (the crush) is .084 seconds and hence the deceleration is $12.57/0.084 = 149.6$ fps squared. The mass of the child is taken as 1.24 slugs (40 pounds), which is the average weight of children of that age (2). Thus, the force on the cg of the child on the outside of the rotation (the child on the left in the picture) is given by $F = ma = 1.24 \times 149.6 = 185.5$ pounds. This is the total force on the child due to the impact.

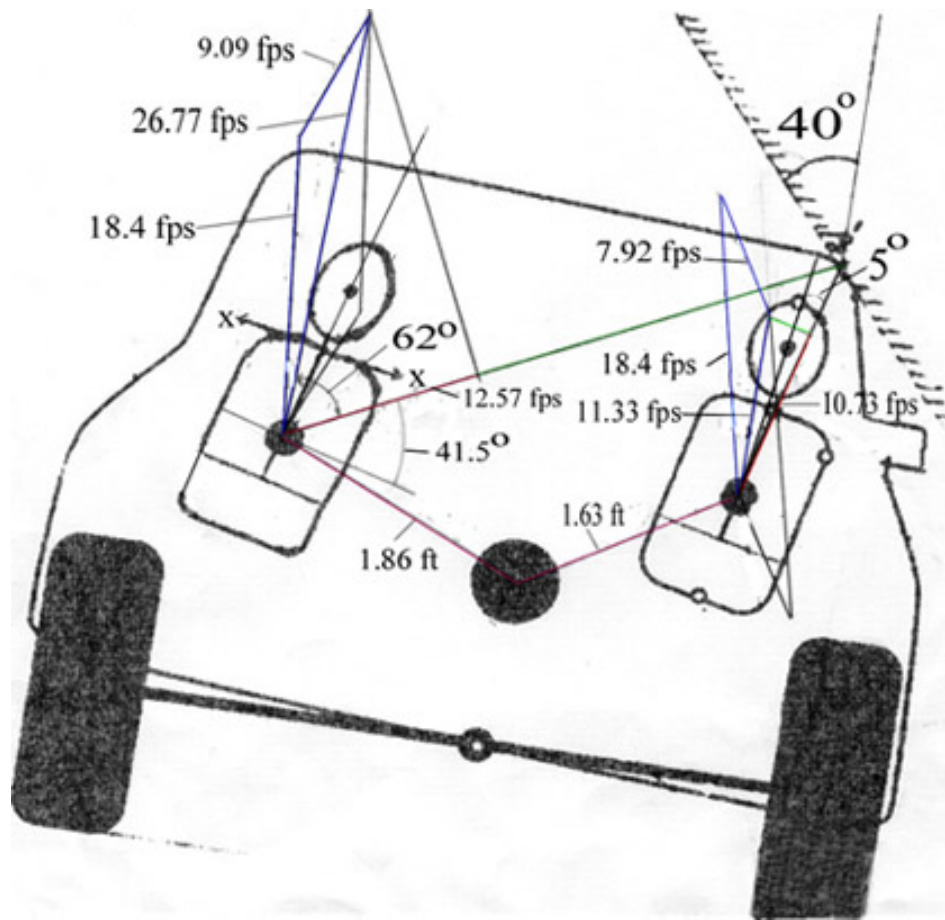


Figure 5

For the child on the inside of the rotation (the child on the right in the picture), we find that the velocity change is 10.73 fps at an angle of 5 degrees to the vertical axis of the child; hence, the deceleration is $10.73/.084 = 127.74$ fps squared and $F = 1.24 \times 127.74 = 158.4$ pounds.

The most common type of booster seat harness for children of that age is the X harness. Thus the force will be divided between the shoulder and the hip. Referring to the nomenclature of figure 6, F is the total force on the child due to the impact, F_1 is the force of the belt at the shoulder, F_2 is the force of the belt at the hip, l_1 is the vertical distance from the center of gravity of the child to the neck, l_2 is the vertical distance from the cg to the hip, β is the angle of the force vector at the cg relative to the direction perpendicular to the vertical axis of the child, β_1 is the corresponding angle at the shoulder, x is the direction parallel to the shoulders (i.e., perpendicular to the neck), y is the direction along the neck (i.e., perpendicular to the shoulder) and M is the moment about the cg of the child. Notice that the belt at the hip is not able to support a vertical force.

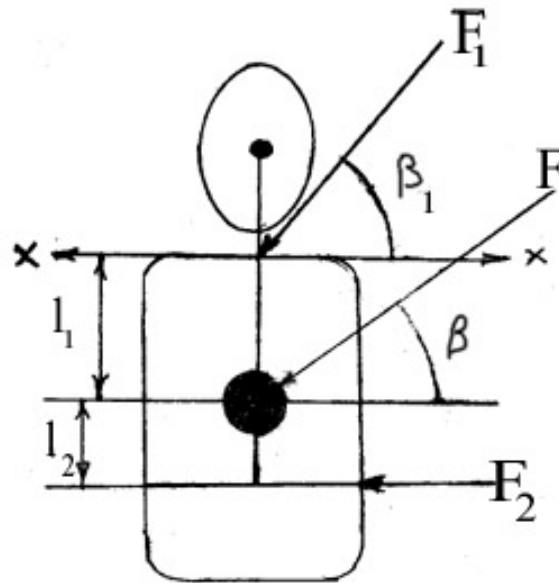


Figure 6

From figure 5, for the outside child we find $\beta = 41.5$ degrees, and for the inside child we find $\beta = 85$ degrees. Summing the forces in the x and y directions and the moments about the cg for each child, we get:

$$\sum F_x = F_1 \cos \beta_1 + F_2 = F \cos \beta$$

$$\sum F_y = F \sin \beta = F_1 \sin \beta_1$$

$$\sum M = F_1 l_1 \cos \beta_1 - F_2 l_2 = 0$$

This gives us three equations in three unknowns. We may most conveniently solve these equations by noting that

$$F_2 = F_1 \frac{l_1}{l_2} \cos \beta$$

$$F_1 \left(1 + \frac{l_1}{l_2}\right) \cos \beta_1 = F \cos \beta$$

$$F \sin \beta = F_1 \sin \beta_1$$

$$F \cos \beta = F_1 \left(1 + \frac{l_1}{l_2}\right) \cos \beta_1$$

$$\text{Hence, } \tan \beta_1 = \left(1 + \frac{l_1}{l_2}\right) \tan \beta$$

We take l_1 as 10.2 inches and l_2 as 3.4 inches. Solving for the outside child, we find $F_1 = 128.63$ pounds, $\beta_1 = 74.22^\circ$, $F_{1x} = 34.98$ pounds $= F_1 \cos \beta_1$, $F_2 = 104.94$ pounds, and $F_{1y} = 123.78$ pounds $= F_1 \sin \beta_1$. For the child on the inside of the rotation we find $F_1 = 157.8$ pounds, $F_{1x} = 3.46$ pounds, $F_2 = 10.37$ pounds, $F_{1y} = 157.76$ pounds and $\beta_1 = 88.75^\circ$.

These numbers neglect the whiplash effect on the head, which would make the x component of the forces slightly lower for the outside child, but would cause additional injuries to the neck.

References:

1. Yamaguchi, G. T., et al, Development of a Computational Method to Predict Occupant Motions and Neck Loads During Rollovers. SAE Technical Paper Series, 2005-01-0300.
2. Weights and Dimensions of Children. United States Center for Disease Control, CDC, www.cdc.gov <<http://www.cdc.gov/>>.
3. Picture Gallery on the Safety Choice Coalition website, www.safetychoice.org <<http://www.safetychoice.org/>>.
4. Larsen, R. E., et al, Vehicle Rollover Testing, Methodologies in Recreating Rollover Collisions. SAE Technical Paper Series, 2000-01-1641.